Spin decoherence by spacetime curvature

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Abstract
A mechanism of spin decoherence caused by spacetime curvature in general
relativity is discussed. The spin state of a particle is shown to decohere only if
the particle moves in a curved spacetime. In particular, when a particle is near
the event horizon of a black hole, an extremely rapid spin decoherence occurs
for an observer who is static in a Killing time, however slow the particle's
motion.

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1. Introduction

In quantum information processing [1], the spin of a spin-1/2 particle is often utilized as a
qubit regardless of the momentum state of the particle. However, the spin and momentum
are not, in general, separable in the relativistic regime, since the Lorentz transformation (LT)
entangles them via the Wigner rotation [2]. Among recent work [3–8] on entanglement in
relativity, Peres *et al* showed that the spin entropy of a spin-1/2 particle is not invariant under
the LT unless the particle is in the momentum eigenstate [3]. This means that even if the spin
is in a pure state in one frame of reference, it may not be so in another unless the entanglement
with the momentum is taken into account. Thus, the spin alone cannot be used as a qubit in
quantum information processing that involves relativistically moving observers.

In this paper, we address the question of how the spin of a particle moving in a gravitational
field decoheres due to the effects of general relativity. In general relativity, a gravitational field
is represented by a spacetime curvature, which entails a breakdown of the global rotational
symmetry. The spin in general relativity can therefore be defined only locally by invoking the
rotational symmetry of the local inertial frame. As a consequence of this local definition, the
motion of the particle is accompanied by a continuous succession of LTs. We shall show that
this effect, which is caused by spacetime curvature, gives rise to a spin entropy production
that is unique to general relativity. This means that even if the state of the spin is pure at one
spacetime point, it, in general, becomes mixed at an other spacetime point. As an illuminating
example, we shall show that the spin entropy of a circularly moving particle changes extremely

rapidly near the event horizon of a black hole in a local inertial frame that is static with respect to a Killing time at each point.

It is in order here to remark upon two important distinctions between the present general-relativistic problem and the special-relativistic one discussed in [3–7]. First, while spin entropy is not invariant under the LT in special relativity [3], it *is* invariant under the general coordinate transformation (GCT) in general relativity. The spin is defined relative to the local inertial frame, which is physically determined by the spacetime distribution of matter, whereas the GCT is an artificial relabelling of spacetime points, and therefore does not affect the local inertial frame, leaving the spin state invariant. The transformation that changes the spin entropy is a local LT of the local inertial frame. Second, while in special relativity spin entropy is altered by changing the inertial frame, in general relativity spin entropy can change by a *mere translation* of the particle even though both the general coordinate system and the local inertial frame are fixed at each point; here the spin entropy can be generated because local inertial frames at different points are, in general, different, as shown below. An entropy production of this type cannot be found in other general-relativistic problems that do not involve the spin degrees of freedom [9, 10].

This paper is organized as follows. Section 2 explains our formulation of a relativistic spin in a curved spacetime. Section 3 shows that a spin decoherence is caused by spacetime curvature. Section 4 considers an example in the Schwarzschild spacetime. Section 5 summarizes our results.

2. Formulation

A gravitational field in general relativity is described by a curved spacetime with metric $g_{\mu\nu}(x)$. To discuss the spin of a particle in the curved spacetime, we introduce a local inertial frame at each point. The coordinate transformation from a general coordinate system x^μ to the local inertial frame x^a at each point can be carried out using a vierbein (or a tetrad) $e_a{}^\mu(x)$ defined by [11]

$$e_a{}^\mu(x)e_b{}^\nu(x)g_{\mu\nu}(x) = \eta_{ab}, \tag{1}$$

where $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$ is the Minkowski metric. Here and henceforth, it is assumed that Latin and Greek letters run over the four inertial-coordinate labels $0, 1, 2, 3$ and the four general-coordinate labels, respectively, and that repeated indices are to be summed. The vierbein transforms a tensor in a general coordinate system x^μ into that in a local inertial frame x^a, and vice versa. For example, momentum $p^\mu(x)$ in the general coordinate system can be transformed into that in the local inertial frame at x^μ via the relation $p^a(x) = e^a{}_\mu(x)p^\mu(x)$. The inverse of the vierbein $e^a{}_\mu(x)$ is defined by

$$e^a{}_\mu(x)e_a{}^\nu(x) = \delta_\mu{}^\nu, \qquad e^a{}_\mu(x)e_b{}^\mu(x) = \delta^a{}_b. \tag{2}$$

The choice of the local inertial frame is not unique, since the inertial frame remains inertial under the LT. The choice of the vierbein therefore has the same degree of freedom known as the local LT. It is this degree of freedom that transforms the spin of a particle. Namely, a spin-1/2 particle in the curved spacetime is defined as a particle whose one-particle states furnish the spin-1/2 representation of the local LT, not of the GCT. Note that the Dirac field in the curved spacetime is spinor under the local LT, whereas it is scalar under the GCT. Usually, the definition of a particle is not unique in quantum field theory in curved spacetime [12], because we cannot uniquely choose the time coordinate to define the positive energy. However, in the present formulation, our particle is specified by the choice of the vierbein, since $e_0{}^\mu(x)$ generates a preferred global time coordinate from the local inertial time coordinate (the 0-axis).

Consider a wave packet of a spin-$1/2$ particle with mass m in the curved spacetime. We assume that the spacetime curvature does not change drastically within the spacetime scale of the wave packet that represents a state of a particle. Let us suppose that the centroid of the wave packet is located at point x^μ and is moving with four-velocity $dx^\mu/d\tau = u^\mu(x)$, which is normalized as $u^\mu(x)u_\mu(x) = -c^2$; this motion is not necessarily geodesic in the presence of an external force. The momentum of the centroid is then given by $q^a(x) = e^a_{\ \mu}(x)(mu^\mu(x))$ in the local inertial frame at the point x^μ. Moreover, using this local inertial frame, we can describe the wave packet as in special relativity. Namely, the momentum eigenstate $|p^a, \sigma\rangle$ of the particle is labelled by the four-momentum $p^a = (\sqrt{|\vec{p}|^2 + m^2c^2}, \vec{p})$ and by the z-component σ $(=\uparrow, \downarrow)$ of the spin [13]. The wave packet is then expressed as a linear combination of $|p^a, \sigma\rangle$,

$$|\psi\rangle = \sum_\sigma \int d^3\vec{p}\, N(p^a)C(p^a, \sigma)|p^a, \sigma\rangle, \tag{3}$$

where

$$d^3\vec{p}\, N(p^a) \equiv d^3\vec{p}\, \frac{mc}{\sqrt{|\vec{p}|^2 + m^2c^2}} \tag{4}$$

is a Lorentz-invariant volume element. From the normalization condition

$$\langle p'^a, \sigma'|p^a, \sigma\rangle = \frac{1}{N(p^a)}\delta^3(\vec{p}' - \vec{p})\delta_{\sigma'\sigma}, \tag{5}$$

we find that the coefficient $C(p^a, \sigma)$ satisfies

$$\sum_\sigma \int d^3\vec{p}\, N(p^a)|C(p^a, \sigma)|^2 = 1. \tag{6}$$

Taking the trace of the density matrix $\rho = |\psi\rangle\langle\psi|$ over the momentum, we obtain the reduced density matrix for the spin,

$$\rho_r(\sigma'; \sigma) = \int d^3\vec{p}\, N(p^a)\langle p^a, \sigma'|\rho|p^a, \sigma\rangle$$

$$= \int d^3\vec{p}\, N(p^a)C(p^a, \sigma')C^*(p^a, \sigma). \tag{7}$$

The spin entropy of the wave packet is the von Neumann entropy of this reduced density matrix:

$$S = -\text{Tr}[\rho_r(\sigma'; \sigma) \log_2 \rho_r(\sigma'; \sigma)]. \tag{8}$$

It is important to note that the spin index σ denotes not the Dirac spin but the Wigner one [2], which is based on the Poincaré symmetry. Using the Pauli matrices $\vec{\sigma}$, Wigner spin operator for the wave packet is given by [14]

$$\hat{\vec{S}} = \frac{1}{2}\sum_{\alpha, \beta} \vec{\sigma}_{\alpha\beta} \int d^3\vec{p}\, N(p^a)|p^a, \alpha\rangle\langle p^a, \beta|, \tag{9}$$

while Dirac spin operator is given by

$$\hat{\vec{\Sigma}} = \int d^3x : \hat{\psi}^\dagger_m(x) \left[\frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}\right]_{mn} \hat{\psi}_n(x) :, \tag{10}$$

where $\hat{\psi}_m(x)$ $(m = 1, 2, 3, 4)$ is a Dirac spinor field and the colons $::$ mean the normal ordering. As is well known, the Dirac spin is not a conserved quantity, since $\hat{\vec{\Sigma}}$ does not commute with the Hamiltonian. In contrast, the Wigner spin is a conserved quantity, since $\hat{\vec{S}}$ does commute with the Hamiltonian. Because the Wigner spin is defined using the particle's rest frame, it is analogous to the non-relativistic spin. Thus, with the Wigner spin, we can discuss a spin observable even in a relativistic regime.

3. Decoherence

After an infinitesimal proper time $d\tau$, the centroid of the wave packet moves to a new point $x'^\mu = x^\mu + u^\mu(x) \, d\tau$, and the wave packet is then described by the local inertial frame at the new point x'^μ. In the new local inertial frame, the momentum of the centroid changes to $q^a(x') = q^a(x) + \delta q^a(x)$ due to an acceleration by the external force *and* due to a change in the local inertial frame. The explicit form of $\delta q^a(x)$ is given by [15]

$$\delta q^a(x) = (ma^a(x) + \chi^a{}_b(x)q^b(x)) \, d\tau, \tag{11}$$

where

$$a^a(x) = e^a{}_\mu(x)(u^\nu(x)\nabla_\nu u^\mu(x)) \tag{12}$$

is the acceleration by the external force and

$$\chi^a{}_b(x) = u^\mu(x)\big(e_b{}^\nu(x)\nabla_\mu e^a{}_\nu(x)\big) \tag{13}$$

is the change in the local inertial frame along $u^\mu(x)$. Since $q^a(x)q_a(x) = -m^2c^2$ and $q^a(x)a_a(x) = 0$, the change $q^a(x) \to q^a(x) + \delta q^a(x)$ may be interpreted as a local LT $\delta^a{}_b + \lambda^a{}_b(x) \, d\tau$, where

$$\lambda^a{}_b(x) = -\frac{1}{mc^2}(a^a(x)q_b(x) - q^a(x)a_b(x)) + \chi^a{}_b(x). \tag{14}$$

While the first term due to the acceleration exists even in special relativity, the second term due to the spacetime curvature is unique to general relativity. Note that even if the wave packet moves as straight as possible (i.e., moves along a geodesic curve $a^a(x) = 0$), this LT may be nonzero in general relativity. By iterating this infinitesimal transformation, we obtain a transformation formula for a finite proper time as a time-ordered Dyson series, since $\lambda^a{}_b(x)$ at different points do not necessarily commute. When the centroid of the wave packet moves along a path $x^\mu(\tau)$ from $x_i^\mu = x^\mu(\tau_i)$ to $x_f^\mu = x^\mu(\tau_f)$, the motion of the wave packet is accompanied by a LT given by

$$\Lambda^a{}_b(x_f, x_i) = T \exp\left(\int_{\tau_i}^{\tau_f} \lambda^a{}_b(x(\tau)) \, d\tau\right), \tag{15}$$

where T is the time-ordering operator.

Since the spin entropy is not invariant under the LT, neither is it invariant during the motion of the wave packet. Note that the momentum eigenstate $|p^a, \sigma\rangle$ transforms under a LT $\Lambda^a{}_b$ as [13, 16]

$$U(\Lambda)|p^a, \sigma\rangle = \sum_{\sigma'} D^{(1/2)}_{\sigma'\sigma}(W(\Lambda, p))|\Lambda p^a, \sigma'\rangle, \tag{16}$$

where $D^{(1/2)}_{\sigma'\sigma}(W(\Lambda, p))$ is the 2×2 unitary matrix that represents the Wigner rotation $W^a{}_b(\Lambda, p)$; the explicit form of the Wigner rotation reads

$$W^a{}_b(\Lambda, p) = (L^{-1}(\Lambda p)\Lambda L(p))^a{}_b, \tag{17}$$

where $L^a{}_b(p)$ describes a standard LT defined by

$$L^0{}_0(p) = \gamma, \qquad L^0{}_i(p) = L^i{}_0(p) = p^i/mc,$$
$$L^i{}_k(p) = \delta_{ik} + (\gamma - 1)p^i p^k/|\vec{p}|^2, \tag{18}$$

with $\gamma = \sqrt{|\vec{p}|^2 + m^2c^2}/mc$ and $i, k = 1, 2, 3$. Since the Wigner rotation of the spin depends on the momentum, the LT entangles the spin with the momentum. In accordance with the

LT (15) induced by the motion of the wave packet, the reduced density matrix for the spin in the local inertial frame at x_f^μ becomes

$$\rho_{\mathrm{r}}'(\sigma'; \sigma) = \sum_{\sigma''\sigma'''} \int \mathrm{d}^3\vec{p}\, N(p^a) C(p^a, \sigma'') C^*(p^a, \sigma''') D_{\sigma'\sigma''}^{(1/2)}(W(\Lambda(x_f, x_i), p))$$

$$\times D_{\sigma\sigma'''}^{(1/2)*}(W(\Lambda(x_f, x_i), p)). \qquad (19)$$

This reduced density matrix $\rho_{\mathrm{r}}'(\sigma'; \sigma)$ at τ_f, in general, represents a mixed state, even if the initial reduced density matrix $\rho_{\mathrm{r}}(\sigma'; \sigma)$ represents a pure state at τ_i. This means that the spin entropy is generated by both gravity and acceleration during the motion of the wave packet.

4. Example

As an example, we consider the Schwarzschild spacetime [17],

$$g_{\mu\nu}(x)\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu = -f(r)c^2\, \mathrm{d}t^2 + \frac{1}{f(r)}\, \mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2), \qquad (20)$$

where $f(r) = 1 - (r_s/r)$, with r_s being the Schwarzschild radius. At this radius, the spacetime has the event horizon, on which the coordinate system (t, r, θ, ϕ) breaks down. The time coordinate t is known as a Killing time, with respect to which the Schwarzschild spacetime is static. In the Schwarzschild spacetime, we introduce a static observer with a static local inertial frame at each point by choosing the vierbein (1) as

$$e_0{}^t(x) = \frac{1}{c\sqrt{f(r)}}, \qquad e_1{}^r(x) = \sqrt{f(r)},$$
$$e_2{}^\theta(x) = \frac{1}{r}, \qquad e_3{}^\phi(x) = \frac{1}{r\sin\theta}, \qquad (21)$$

with all the other components being zero. Below, only nonzero components are shown. Note that the inertial frame is defined at each instant, since the observer is accelerated to remain at the given point. Such an accelerated observer would perceive the Hawking radiation [18] if the state of the quantum field is represented in a Fock space which is defined using the Kruskal time. However, in the present example, the observer does not suffer from the Hawking radiation, because we consider a Fock space which is defined using the Killing time. Actually, by the choice of the vierbein (21), the local inertial time coordinate x^0 is parallel to the Killing time coordinate t.

Suppose that the centroid of the wave packet is moving along a circular trajectory of radius r ($> r_s$) with a constant velocity $r\, \mathrm{d}\phi/\mathrm{d}t \equiv v\sqrt{f(r)}$ on the equatorial plane $\theta = \pi/2$. The four-velocity of the centroid is then given by

$$u^t(x) = \frac{\cosh\xi}{\sqrt{f(r)}}, \qquad u^\phi(x) = \frac{c\sinh\xi}{r}, \qquad (22)$$

where ξ is the rapidity in the local inertial frame defined by

$$\tanh\xi = \frac{v}{c}. \qquad (23)$$

Accordingly, in the local inertial frame at any point, the four-momentum of the centroid becomes $q^a(x) = (mc\cosh\xi, 0, 0, mc\sinh\xi)$. We assume that the coefficient (3) of the wave packet at τ_i is

$$C(p^a, \sigma) = \frac{\delta_{\sigma,\uparrow}}{\sqrt{\pi^{1/2} w N(p^a)}} \exp\left(-\frac{(p^3 - q^3(x))^2}{2w^2}\right) \sqrt{\delta(p^1)\delta(p^2)}, \qquad (24)$$

which is Gaussian in p^3 with the spin z-component $\uparrow$, even though p^1 and p^2 have the definite value 0. The spin entropy of this wave packet is zero at time τ_i, since the momentum is not entangled with the spin.

Since the wave packet is not in a geodesic motion, the first term in the infinitesimal LT (14) is not zero in this example, giving rise to a generalized Thomas precession [15] in the curved spacetime. This effect is an origin of the spin–orbit coupling and contributes to the spin entropy even in the limit of Minkowski spacetime $r_s \to 0$. In contrast, the second term in equation (14), which consists of a boost along the 1-axis

$$\chi^0_{\ 1}(x) = \chi^1_{\ 0}(x) = -\frac{cr_s \cosh\xi}{2r^2\sqrt{f(r)}} \tag{25}$$

and a rotation about the 2-axis

$$\chi^1_{\ 3}(x) = -\chi^3_{\ 1}(x) = \frac{c\sinh\xi\sqrt{f(r)}}{r}, \tag{26}$$

causes the spin decoherence that is unique to general relativity. In fact, in the limit of Minkowski spacetime, this term is reduced to a pure rotation, which does not change the spin entropy as shown below.

Combining these terms, we see that the infinitesimal LT $\lambda^a_{\ b}(x)$ becomes

$$\lambda^0_{\ 1}(x) = \lambda^1_{\ 0}(x) = -L\tanh\xi, \tag{27}$$

$$\lambda^1_{\ 3}(x) = -\lambda^3_{\ 1}(x) = L, \tag{28}$$

where

$$L = \frac{c\cosh^2\xi\sinh\xi}{r}\left[1 - \frac{r_s}{2rf(r)}\right]\sqrt{f(r)}. \tag{29}$$

After a proper time $\tau_p = \tau_f - \tau_i$ of the particle, the Wigner rotation (17) for the finite LT (15) is reduced to a rotation about the 2-axis through angle $\Omega(p^a)\tau_p$, where

$$\Omega(p^a) = \left(1 - \frac{p^3}{p^0 + mc}\tanh\xi\right)L. \tag{30}$$

The first and second terms in $\Omega(p^a)$ correspond to the rotation part (28) and the boost part (27), respectively. The corresponding unitary representation can be expressed in terms of the Pauli matrix σ_y as

$$D^{(1/2)}_{\sigma'\sigma}(W(\Lambda(x_f, x_i), p)) = \exp\left(-i\frac{\sigma_y}{2}\Omega(p^a)\tau_p\right). \tag{31}$$

Therefore, the reduced density matrix at τ_f becomes

$$\rho'_r(\sigma'; \sigma) = \frac{1}{2}\begin{pmatrix} 1 + \overline{\cos\Omega\tau_p} & \overline{\sin\Omega\tau_p} \\ \overline{\sin\Omega\tau_p} & 1 - \overline{\cos\Omega\tau_p} \end{pmatrix}, \tag{32}$$

where the overline means the average over the momentum distribution, $\overline{X} = \int d^3\vec{p}\, N(p^a)|C(p^a, \uparrow)|^2 X(p^a)$.

As a result, the spin entropy generated by the gravity and acceleration is given by

$$S' = -P\log_2 P - (1 - P)\log_2(1 - P), \tag{33}$$

where

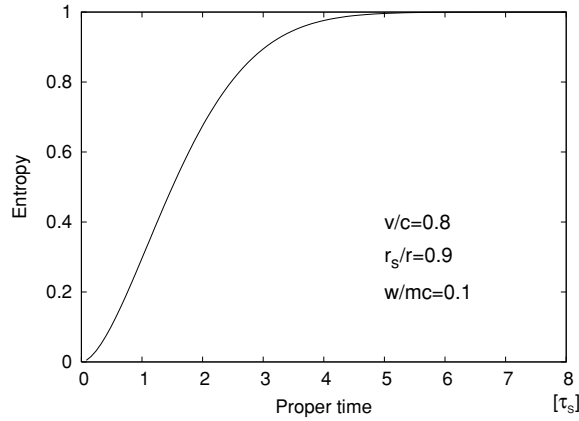$$P = \tfrac{1}{2}(1 - |\overline{\exp(i\Omega\tau_p)}|). \tag{34}$$

Figure 1. The generated entropy S' as a function of the proper time τ_p at v/c ($=\tanh\xi$) $= 0.8$, $r/r_s = 0.9$ and $w/mc = 0.1$. The proper time is normalized by $\tau_s = mr_s/w$.

Note that only the p^a-dependent term in $\Omega(p^a)$, which arises from the boost part (27), contributes to the spin entropy. It is easy to see that $0 \leqslant P \leqslant 1/2$ and $0 \leqslant S' \leqslant 1$, since

$$0 \leqslant \overline{|\exp(i\Omega\tau_p)|^2} \leqslant \overline{|\exp(i\Omega\tau_p)|^2} = 1. \tag{35}$$

If the wave packet is in the momentum eigenstate with $w = 0$ of equation (24), spin entropy is not generated, i.e. $S' = 0$, since then the second inequality in equation (35) is saturated.

In the case of $w/mc \ll 1$, expanding $\Omega(p^a)$ around $q^3(x) = mc \sinh\xi$ up to the second order in $(p^3 - q^3(x))/mc$, we obtain

$$\overline{|\exp(i\Omega\tau_p)|} \simeq \frac{1}{\left(1 + A^2\tau_p^2\right)^{1/4}} \exp\left(-\frac{B^2\tau_p^2}{4\left(1 + A^2\tau_p^2\right)}\right), \tag{36}$$

where

$$A = \frac{Lw^2\tanh^2\xi}{2m^2c^2}\left(\frac{1}{(\cosh\xi + 1)^2} - \frac{1}{\cosh^2\xi}\right), \tag{37}$$

$$B = \frac{Lw\tanh\xi}{mc}\left(\frac{1}{\cosh\xi} - \frac{1}{\cosh\xi + 1}\right). \tag{38}$$

We thus find that the reduced density matrix decoheres to a mixed state $S' > 0$ after the proper time $\tau_p \sim |B|^{-1}$ and becomes maximally mixed ($S' \to 1$) in the limit of $\tau_p = \infty$, as shown in figure 1. No revival of coherence occurs as opposed to the case of an electron in an atom. In this latter case the electron has a discrete spectrum and no real decoherence occurs, while in our case the momentum distributes continuously due to a semiclassical prescription in which the orbital motion is fixed to a given one by an appropriate external force at a macroscopic radius. Figure 2 shows the inverse of the characteristic decoherence time, i.e. $|B|$, as a function of r_s/r. The spin decoherence is extremely rapid near the event horizon, because $|L|$ becomes very large as r approaches r_s. We emphasize that this is true even when the wave packet moves with non-relativistic velocity, if r is sufficiently close to r_s or if w is sufficiently large. In particular, right on the event horizon, the reduced density matrix becomes maximally mixed in an infinitesimal proper time. Of course, the formula (33) no longer holds inside the event horizon $r < r_s$, since the strong gravity makes it impossible for the particle to move circularly. At the spatial infinity $r \to \infty$, on the other hand, the reduced density matrix remains pure
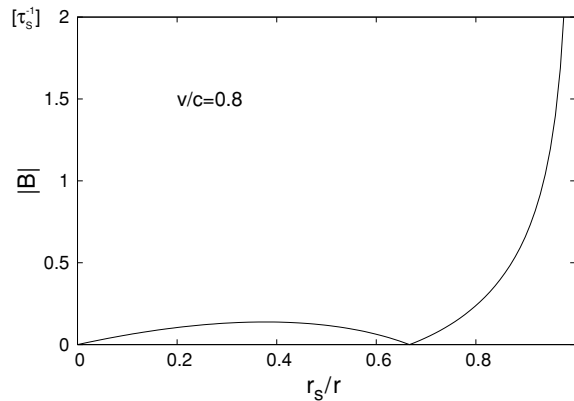
Figure 2. The inverse of the characteristic decoherence time, $|B|$, as a function of r_s/r at $v/c = 0.8$, normalized by $\tau_s^{-1} = w/mr_s$. $|B| = 0$ at $r = \infty$ or $3r_s/2$, whereas $|B| = \infty$ at $r = r_s$.

($S' = 0$), since both gravity and acceleration vanish there. An interesting situation occurs on the sphere of the radius $r = 3r_s/2$, on which the reduced density matrix does not decohere because the LT caused by gravity is cancelled by that caused by acceleration, giving $L = 0$.

This spin decoherence is very slow in the gravitational field of the earth, where $r_s \sim 1$ cm. For example, when a wave packet of a particle with mass m and momentum spread w (see equation (24)) is at rest in the International Space Station (ISS) going around the earth with $v \sim 7.7$ km s^{-1} and $r \sim 6800$ km, the characteristic decoherence time $|B|^{-1}$ is about $2.2 \times mc/w$ years.

In contrast to the static observer, an observer moving with the wave packet does not see the spin decoherence, because $\delta q^a(x)$ in equation (11) is zero in the co-moving local inertial frame. However, these two results do not contradict each other, since the spin entropy is not invariant under the LT from a static local inertial frame to a co-moving one.

5. Conclusion

In conclusion, we have shown that spin entropy is generated when a spin-1/2 particle moves in a gravitational field. Even if the spin at one spacetime point is in a pure state, it may evolve into a mixed state as the particle moves. In particular, the spin entropy of a circularly moving particle increases very rapidly near the event horizon of the Schwarzschild black hole. Unless the entanglement with the momentum is taken into account, the spin cannot be used as a qubit in quantum information processing where the system is subject to a strong gravitational field.

Acknowledgments

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